Economic Equivalence

Lecture 5
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• What do we mean by “economic equivalence?”

• Why do we need to establish an economic equivalence?

• How do we establish an economic equivalence?
• Economic equivalence exists between cash flows that have the same economic effect and could therefore be traded for one another.
Combination of interest rate (rate of return) and time value of money to determine different amounts of money at different points in time that are economically equivalent
Example of Equivalence

Different sums of money at different times may be equal in economic value at a given rate.

$100 now is economically equivalent to $110 one year from now, if the $100 is invested at a rate of 10% per year.
**Example**: You borrowed $5,000 from a bank and you have to pay it back in 5 years. There are many ways the debt can be repaid. \((i = 0.08)\)

*Plan 1*: At end of each year pay $1,000 principal plus interest due.

*Plan 2*: Pay interest due at end of each year and principal at end of five years.

*Plan 3*: Pay in five end-of-year payments ($1,252).

*Plan 4*: Pay principal and interest in one payment at end of five years.

All these plans are equivalent in the sense that the sum of all outgoing cash flows at time 0 is $5,000.
Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal.
• If you deposit $P$ dollars today for $N$ periods at $i$, you will have $F$ dollars at the end of period $N$.

\[ F = P(1+i)^N \]
Example

At what interest rate would these two amounts be equivalent?

$i = ?$

$\$2,042$

$\$3,000$

0

5
Practice Problem

At 8% interest, what is the equivalent worth of $2,042 now 5 years from now?

If you deposit $2,042 today in a savings account that pays 8% interest annually, how much would you have at the end of 5 years?

$2,042

F
Solution

\[ F = \$2,042 \times (1 + 0.08)^5 \]

\[ = \$3,000 \]
Compute the equivalent lump-sum amount at $n = 3$ at 10% annual interest.
Approach

$100  $80  $120  $150  $200  $100
$V_3 = $511.90 + $264.46 = $776.36

$100(1+0.10)^3 + $80(1+0.10)^2 + $120(1+0.10) + $150 = $511.90

$200(1+0.10)^{-1} + $100(1+0.10)^{-2} = $264.46
At what interest rate would you be indifferent between the two cash flows?
Approach

- Step 1: Select the base period to compute the equivalent value (say, $n = 3$)

- Step 2: Find the net worth of each at $n = 3$. 
Establish Equivalence at $n = 3$

Option A: $F_3 = 500(1 + i)^3 + 1,000$

Option B: $F_3 = 502(1 + i)^2 + 502(1 + i) + 502$

- Find the solution by trial and error, say $i = 8%$

Option A: $F_3 = 500(1.08)^3 + 1,000$

$= 1,630$

Option B: $F_3 = 502(1.08)^2 + 502(1.08) + 502$

$= 1,630$
• Economic equivalence is used commonly in engineering to compare alternatives.

• In engineering economy, two things are said to be equivalent if they have the same effect.

• Unlike most individuals involved with personal finances, corporate and government decision makers using engineering economics might not be so much concerned with the timing of a project's cash flows as with the profitability of the project.
Therefore, **analytical tools** are needed to compare projects involving receipts and disbursement occurring at different times, with the goal of identifying an alternative having the largest eventual profitability.
Equivalence Calculations

• Two cash flows need to be presented along the same time period using a similar format to facilitate comparison.

• When interest is earned, monetary amounts can be directly added only if they occur at the same point in time.

• Equivalent cash flows are those that have the same value.
Interest rate

- An interest rate is the rate at which interest is paid by borrowers for the use of money that they borrow from a lender.

- For example, a small company borrows capital from a bank to buy new assets for its business, and in return the lender receives interest at a predetermined interest rate.
• Interest rates are normally expressed as a percentage of the principal for a period of one year.

**IMPORTANT:**

• The central banks or reserve banks of countries generally tend to reduce interest rates when they wish to increase investment and consumption in the country's economy.
• However, a low interest rate as a macro-economic policy can be risky and may lead to the creation of an economic bubble, in which large amounts of investments are poured into the real-estate market and stock market.
The Interest Rate

Which would you prefer -- $10,000 today or $10,000 in 5 years?

Obviously, $10,000 today.

You already recognize that there is

TIME VALUE TO MONEY!!
Why TIME?

Why is **TIME** such an important element in your decision?

**TIME** allows you the *opportunity* to postpone consumption and earn **INTEREST**.
Types of Interest

• Simple Interest
Interest paid (earned) on only the original amount, or principal, borrowed (lent).

• Compound Interest
Interest paid (earned) on any previous interest earned, as well as on the principal borrowed (lent).
Simple Interest Formula

Formula

\[ \text{SI} = P_0(i)(n) \]

SI: Simple Interest

\( P_0 \): Deposit today \((t=0)\)

\( i \): Interest Rate per Period

\( n \): Number of Time Periods
Simple Interest Example

• Assume that you deposit $1,000 in an account earning 7% simple interest for 2 years. *What is the accumulated interest at the end of the 2nd year?*

\[
SI = P_0(i)(n) \\
= \$1,000(.07)(2) \\
= \$140
\]
• **Future Value** is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate.
Simple Interest (FV)

- What is the Future Value (FV) of the deposit?

\[
FV = P_0 + SI \\
= $1,000 + $140 \\
= $1,140
\]
• **Present Value** is the current value of a future amount of money, or a series of payments, evaluated at a given interest rate.
Simple Interest (PV)

- What is the Present Value (PV) of the previous problem?

The Present Value is simply the $1,000 you originally deposited. That is the value today!
Future Value
Single Deposit (Formula)

\[ FV_1 = P_0 (1+i)^1 \]

\[ = P_0 (1+0.07) \]

\[ = \$1,000 (1.07) \]

\[ = \$1,070 \]

**Compound Interest**

You earned $70 interest on your $1,000 deposit over the first year.

This is the same amount of interest you would earn under simple interest.
Future Value
Single Deposit (Formula)

\[ FV_1 = P_0 (1+i)^1 \]
\[ = \$1,000 \times 1.07 \]
\[ = \$1,070 \]

\[ FV_2 = FV_1 (1+i)^1 \]
\[ = P_0 (1+i)(1+i) \]
\[ = P_0 (1+i)^2 \]
\[ = \$1,000 \times 1.07 \times 1.07 \]
\[ = \$1,000 \times 1.07^2 \]
\[ = \$1,144.90 \]

You earned an EXTRA $4.90 in Year 2 with compound over simple interest.
General Future Value Formula

\[ FV_1 = P_0(1+i)^1 \]
\[ FV_2 = P_0(1+i)^2 \]

etc.

General Future Value Formula:
\[ FV_n = P_0 (1+i)^n \]
General Present Value Formula

\[ PV_0 = \frac{FV_1}{(1+i)^1} \]
\[ PV_0 = \frac{FV_2}{(1+i)^2} \]

etc.

General Present Value Formula:

\[ PV_0 = \frac{FV_n}{(1+i)^n} \]
Example

- Find the PV of $500 to be received in 5 years, with:
- 12% stated annual rate, \textit{annual compounding},

\[ PV = \frac{500}{(1 + .12)^5} = \$283.71 \]

- 12% stated annual rate, \textit{semiannual compounding},

\[ PV = \frac{500}{(1 + (.12 / 2))^{10}} = \$279.20 \]

- 12% stated annual rate, \textit{quarterly compounding},

\[ PV = \frac{500}{(1 + (.12 / 4))^{20}} = \$276.84 \]
Elasticity

- Price elasticity of demand is a measure used in economics to show the responsiveness, or elasticity, of the quantity demanded of a good or service to a change in its price.

- More precisely, it gives the percentage change in quantity demanded in response to a one percent change in price.
Figure 1

12.5% change \((9-8)/8\)

25% change \((4-3)/4\)
Figure 2

50% change \( \frac{12-8}{8} \)

25% change \( \frac{4-3}{4} \)
• **Price Elasticity of Demand**: The responsiveness of quantity demanded to a change in price.

• **Price Elasticity of Supply**: The responsiveness of quantity supplied to a change in price.

• **Income Elasticity of Demand**: The responsiveness of quantity demanded to a change in income.

• **Cross Price Elasticity of Demand**: The responsiveness of quantity demanded of one good to a change in the price of another good.
The Mathematical Representation of Elasticity

Elasticity = \frac{\% \Delta Q}{\% \Delta P}

Because the demand curve is downward sloping and the supply curve is upward sloping the elasticity of demand is negative and the elasticity of supply is positive. Often these signs are implicit and ignored.
Present Worth Analysis

- Describing Project Cash Flows
- Initial Project Screening Method
- Present Worth Analysis
- Variations of Present Worth Analysis
- Comparing Mutually Exclusive Alternatives
Bank Loan vs. Investment Project

**Bank Loan**
- Bank
  - Loan to Customer
  - Repayment

**Investment Project**
- Company
  - Investment to Project
  - Return